Understand the Classification of Numbers for WBCS Prelims Exam

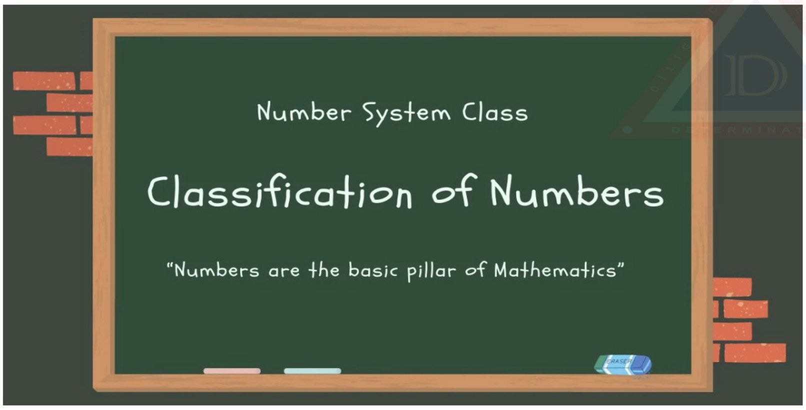
Numbers are the basic pillar of Mathematics, the language of the universe. The thing which we learned in our first mathematics class. The thing we deal with, in our day-to-day life. The thing which is almost everywhere.

Well, we are not here to talk about the universe. But let me make this thing very clear. Classification of Numbers is the most important topic of the quant section. Not only this one, but the entire "Number System". But I promise you, I will make this chapter most interesting and a favorite one.

So, just be with me in the entire session and see how these numbers are going to amaze you every time. So, be ready to enjoy as well as learn.

Overview of the Classification of Numbers

The classification of numbers is a fundamental concept in mathematics, serving as the building block for a wide range of mathematical disciplines. This overview delves into the various categories of numbers, including natural numbers, integers, rational numbers, irrational numbers, and real numbers.



Understanding these classifications is pivotal in mathematics and its applications. Whether you're a student seeking a strong foundation in math or a curious learner exploring the beauty of numbers, this guide will provide you with a comprehensive insight into the world of mathematical classification.

Here is the chart which classifies numbers.

Asutosh Mukherjee Road, College Para, Hathi More, Siliguri, West Bengal
+91 - 8436586516, +91-8436900456
thedhronas@gmail.com
thedronas.com

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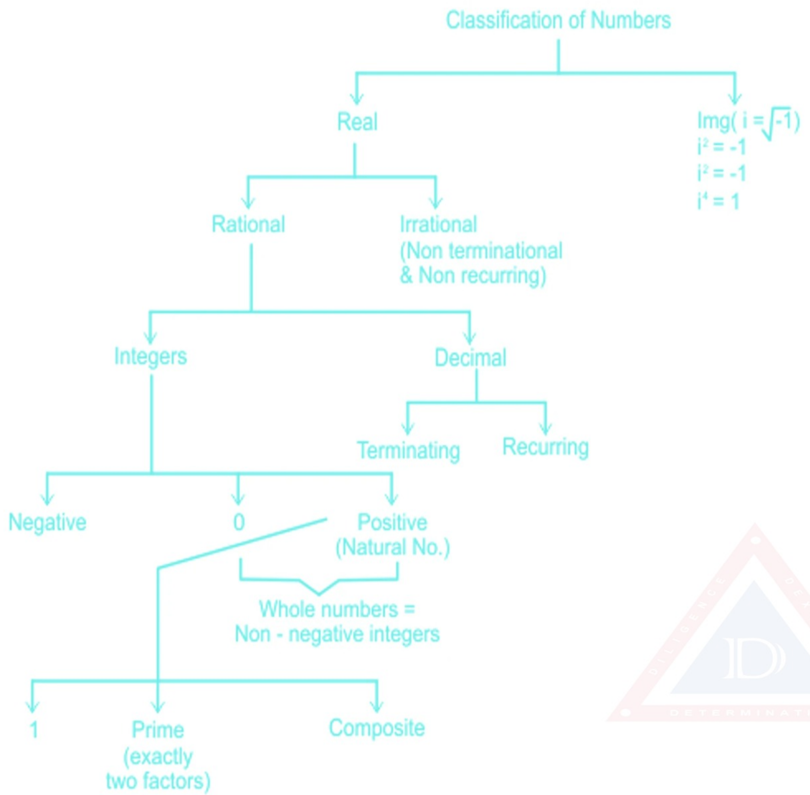

Now that you've taken a look at the overview, let's understand what all these numbers mean.

Real Numbers

Real numbers are the numbers that we use to represent quantities in the real world. These are the numbers whose squares are positive numbers. They include all the rational numbers, such as integers and fractions, as well as the irrational numbers, such as pi and the square root of 2.

Real numbers can be thought of as all the points on a number line. The points corresponding to integers are equally spaced, but the points corresponding to irrational numbers are not.

Real numbers are used in many different areas of mathematics, including arithmetic, algebra, geometry, and calculus. They are also used in many different scientific disciplines, such as physics, chemistry, and engineering.

Here are some examples of real numbers:

- **Integers:** 1, 2, 3, -4, -5, ...
- **Fractions:** 1/2, 2/3, 3/4, -4/5, -5/6, ...
- **Irrational numbers:** π, √2, e, ...

We will discuss all these examples in detail.

Some of the important properties of real numbers include:

- **Order:** Real numbers are ordered, meaning that for any two real numbers, a and b, either a is less than b, a is greater than b, or a is equal to b.
- **Arithmetic operations:** Real numbers are closed under the arithmetic operations of addition, subtraction, multiplication, and division. This means that the sum, difference, product, and quotient of any two real numbers is also a real number.
- **Density:** Real numbers are dense, meaning that between any two real numbers, there is always another real number.
- **Completeness:** Real numbers are complete, meaning that every non-empty set of real numbers that has an upper bound has a least upper bound.

Real numbers are an essential part of mathematics and science. They allow us to represent and measure quantities in the real world, and to solve complex problems.

Imaginary Numbers

Imaginary numbers are numbers that are not real numbers. They are defined as multiples of the imaginary unit **i**, which is defined by the property $\mathbf{i^2 = -1}$.

[$i = \sqrt{(-1)}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$]

e.g. If the number 6i is given, it means it's value is $6 \times \sqrt{(-1)} = 6\sqrt{(-1)} = \sqrt{(-36)}$.

Imaginary numbers are often used in mathematics and physics to represent quantities that cannot be represented by real numbers. For example, the square root of a negative number is an imaginary number.

Imaginary numbers can be added, subtracted, multiplied, and divided like real numbers. However, there are some important differences between the two types of numbers. For example, the product of two imaginary numbers is a real number, and the quotient of two imaginary numbers is a real or imaginary number, depending on the values of the two numbers.

Imaginary numbers can be represented on a complex plane, which is a two-dimensional plane with a real axis and an imaginary axis. The real part of an imaginary number is plotted on the real axis, and the imaginary part of the imaginary number is plotted on the imaginary axis.

Now that, we've an idea about what the real and imaginary numbers are. Let's understand the types of real numbers.

Real numbers are of two types. i.e.

1. Rational Numbers
2. Irrational Numbers

So let's start with the rational numbers first.

Rational Numbers

The numbers which can be written in the form p/q where p and q are co-prime integers and q ≠ 0. i.e. In decimal form, they are either terminating or recurring numbers. For example, 0.56, 0.45454545…., 4/11, all are rational numbers.

Let's find out how we can convert a rational number from decimal form to fraction form.

e.g. Express 0.82828282…. in the form of a fraction.

Ans: Let x = 0.82828282… ______(1)

As the period containing 2 digits, we multiply by 10^2 = 100,

∴ 100x = 82.82828282…. _______(2)

Now, (2) – (1), we get,

⇒ 99x = 82

⇒ x = 82/99

Hence, 0.82828282 can be written as 82/99.

Let's look at one more example.

e.g. Express 0.024024024024024024….. in the form of a fraction.

Ans: Let x = 0.024024024024…. _____(1)

As the period is containing 3 digits, we multiply with 10^3 = 1000

∴ 1000x = 24.02404024…. _____(2)

Now, (2) – (1), we get,

⇒ 999x = 24

⇒ x = 24/999 = 8/333.

Hence, 0.024024024024... can be written as 8/333.

Rational numbers can be further classified into two types:

Asutosh Mukherjee Road, College Para, Hathi More, Siliguri, West Bengal
+91 - 8436586516, +91-8436900456
thedhronas@gmail.com
thedronas.com

1. Integers
2. Decimals

We will discuss it further in detail. But for now, let's look at the other type of the real numbers i.e. **Irrational Numbers**.

Irrational Numbers

Irrational numbers are real numbers that cannot be expressed as a ratio of two integers. They are often said to have decimal representations that never end or repeat.

In other words, the numbers which cannot be written as p/q are Irrational Numbers. So, in decimal form, these numbers are non-terminating and non-recurring numbers. Some examples of irrational numbers include π, √2, and e.

Here are some of the properties of irrational numbers:

- They cannot be expressed as a ratio of two integers.
- Their decimal representations never end or repeat.
- They are dense on the number line, meaning that between any two irrational numbers there is always another irrational number.
- They are complete, meaning that every non-empty set of irrational numbers that has an upper bound has a least upper bound.

Irrational numbers are also used in many different scientific disciplines, such as physics, chemistry, and engineering.

For example, π is used to calculate the circumference of a circle, and √2 is used to calculate the length of the diagonal of a square.

Irrational numbers are an important part of mathematics and science. They allow us to solve complex problems and to make accurate measurements of the world around us.

Now, let's get back to the types of rational numbers. We will understand **integers** first.

Integers

An integer is a number that can be written without a fractional component. For example, 21, 4, 0, and -2048 are integers, while 9.75 and √2 are not.

Integers can be positive, negative, or zero. They are the most basic type of number, and they are used in many different areas of mathematics, including arithmetic, algebra, and geometry.

Integers can be represented on a number line, with the positive integers on the right and the negative integers on the left. Zero is located in the middle of the number line.

Asutosh Mukherjee Road, College Para, Hathi More, Siliguri, West Bengal
+91 - 8436586516, +91-8436900456
thedhronas@gmail.com
thedronas.com

Integers can be added, subtracted, multiplied, and divided. They can also be compared to each other using the symbols <, >, and =.

Integers can be further classified as:

1. **Negative Integers:** A negative number is an integer that is less than zero. Negative numbers are often used to represent quantities that are missing or absent, such as a debt or a loss. For example, a temperature of -10 degrees Celsius is 10 degrees Celsius below freezing. Negative numbers are represented by a minus sign (-) in front of the number. For example, -5 is negative five.
2. **Natural Numbers (Positive Integers):** Natural numbers are the integers that are more than zero. They are also known as counting numbers or positive integers. Natural numbers start from 1 and go up to infinity. This means that they include all the whole numbers, but not zero.
3. **0**
4. **Whole Numbers:** Whole numbers are basically natural numbers, including 0. These are also called non-negative integers.

Natural Numbers can also be classified further that we will discuss thoroughly later in this blog. Let's first understand what are **decimals**.

Decimals

A decimal is a number that is written with a decimal point. The decimal point separates the whole number part of the number from the fractional part. For example, the decimal number 3.14159 represents the number three and fourteen hundred fifteen thousandths.

Decimals can be used to represent any real number, including integers, fractions, and irrational numbers. They are a convenient way to write numbers that are too large or too small to be written as fractions.

E.g. 2.332, 0.5, 1.25, 3.14159, -2.3456, 0.00001 are decimal numbers.

Decimals can further be classified as:

1. **Terminating Decimals:** A terminating decimal is a decimal number that has a finite number of digits after the decimal point. In other words, these numbers end after a fixed number of digits. For example, 0.87, 82.25, 9.527, 224.9803, etc. are all terminating decimals.
2. **Non-terminating Recurring Decimals:** A non-terminating recurring decimal is a decimal number that has a digit or sequence of digits that repeats endlessly after the decimal point. For example, 0.33333..., 4.65656565..., and 0.171717... are all non-terminating recurring decimals. Non-terminating recurring decimals can be represented as a fraction with a repeating numerator. For example, the non-terminating recurring decimal 0.33333... can be written as the fraction 1/3 and the non-terminating recurring decimal 0.17171717... can be written as the fraction 17/99.

Asutosh Mukherjee Road, College Para, Hathi More, Siliguri, West Bengal
+91 - 8436586516, +91-8436900456
thedhronas@gmail.com
thedronas.com

So, now we can understand that rational numbers are either integers, terminating decimals or recurring decimals. Hence, those numbers which are neither terminating and non-recurring, are called irrational numbers.

Now let's discuss the Natural Numbers.

Natural Numbers can further be classified as:

1. Prime Numbers
2. Composite Numbers
3. 1

Let's discuss it thoroughly.

Prime Numbers

A prime number is a natural number greater than 1 that has exactly two distinct natural number factors (divisors): 1 and itself. For example, 2, 3, 5, 7, 11, 13, are prime numbers.

From 1 to 50, we have 15 prime factors. From 1 to 100, we have 25 prime factors. And these are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

Prime numbers have some properties like:

1. No integer can divide them completely other than 1 and the number itself (that's why they have only two factors).
2. Every prime number should be in the form of [4n + 1 or 4n – 1] for less than 5 and [6n + 1 or 6n – 1] for more than or equal to 5. But the reverse of this statement is not true. e.g. 25 can be written as 6(4) + 1 but it is not a prime number.
3. Every integer, greater than 1, can be written as the product of prime numbers.
4. 2 is the only even number that is prime. [Even numbers are the positive integers that are divisible by 2 i.e. 2, 4, 6, 8, 10, etc. We will discuss it thoroughly in the next blog]

Since, there is no formula to find out whether a number is prime or not. But for small numbers, we can use this trick:

Take the square root of the number (approximately) and check its divisibility by the prime numbers smaller than its square root.

e.g. Check whether 97 is a prime number or not.

Ans: Square root of 97 will be between 9 and 10. So the prime numbers smaller than 9 are 2, 3, 5 and 7. Since 97 is not divisible by any of these, hence it is a prime number.

e.g. Check whether 91 is a prime number or not.

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thedhronas@gmail.com
thedronas.com

Ans: Square root of 91 will be between 9 or 10. So the prime numbers smaller than 9 are 2, 3, 5 and 7. Since 91 is divided by 7 (13 × 7 = 91). 91 is not a prime number.

We will discuss about prime numbers throughout the 'Number System' series as the entire number system is dependent on the properties of prime numbers. And yeah, number system is the basic pillar of the entire mathematics. So you can now understand the importance of this topic.

Composite Numbers

Composite Number is a natural non-prime number greater than 1 i.e. all the non-prime numbers greater than 1 are composite numbers. These numbers have more than two factors. In other words, we can say, Composite numbers are the product of prime numbers. 4 is the first composite number. Examples are composite numbers are 4, 6, 8, 9, 10, 12 etc.

Now, let's understand the important thing:

1 is neither Prime nor Composite number.

Many students get confused about 1, thinking that 1 is a prime number. But no, 1 is neither prime as it doesn't even have two factors and nor composite as it doesn't have more than two factors, obviously.

In conclusion, the classification of numbers serves as the bedrock of mathematics, offering a structured framework to understand the diverse range of numerical values. From natural numbers, which initiate counting, to rational and irrational numbers that span the mathematical landscape, these classifications form the basis for various mathematical operations and real-world applications.

Whether solving complex equations, analyzing data, or exploring the profound mysteries of number theory, a solid grasp of number classification is indispensable. It is not just a mathematical concept but a fundamental tool that empowers individuals to navigate the quantitative world with confidence and precision. Embracing this knowledge is the key to unlocking a deeper understanding of the mathematical universe and its practical significance.

So, this is all for this blog. We will discuss the Divisibility Rule of numbers in our next blog of this 'Number System' blog series. Till then, keep practicing!

Asutosh Mukherjee Road, College Para, Hathi More, Siliguri, West Bengal
+91 - 8436586516, +91-8436900456
thedhronas@gmail.com
thedronas.com