Learn to find the Last digits of the Numbers

Embark on a journey to numerical precision with 'Learn to Find the **Last Digits** of Numbers,' a tailored resource meticulously crafted for **WBCS Exam** aspirants. In the challenging landscape of competitive examinations, the ability to swiftly and accurately determine the last digits of numbers is a strategic advantage. This comprehensive guide empowers candidates with essential techniques, providing a systematic approach to decoding numerical intricacies. Aspirants will navigate through practical examples, gaining proficiency in handling diverse numerical scenarios.

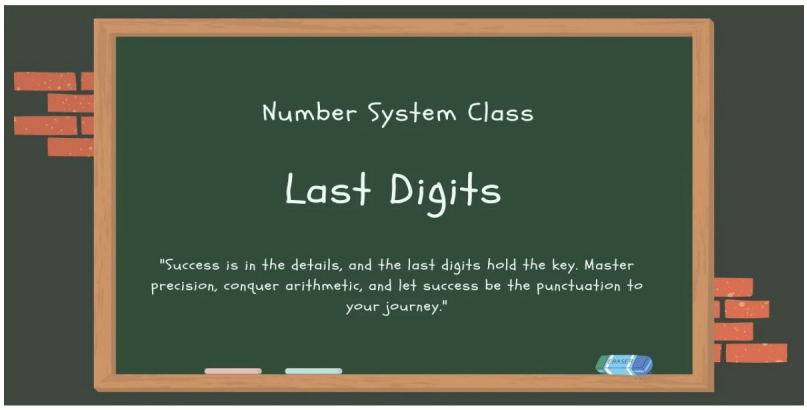
This resource serves as a vital tool, not only for problem-solving efficiency but also for cultivating confidence in tackling arithmetic challenges. Elevate your preparation with this indispensable resource, ensuring a solid foundation for success in the WBCS Exam and enhancing your overall mathematical acumen.

This is the sixth part of the '**Number System**' blog series. So, before understanding concepts of Last Digits, first understand the concept of Remainders by clicking the embedded link. Now, let's start understanding the concepts of **Last Digits**.

Last Digits

We can see in exams some questions like, what will be the last digit (or unit digit) of a number like 2^{2345} or what will be the remainder if 2^{2345} is divided by 10. There is a property of these last digits that, after sometime, these digits start repeating itself in the same pattern in which they occur earlier.

Like, for 2^1, the last digit is 2. For 2^2, the digit is 4. For 2^3, the last digit is 8. For 2^4, the last digit is 6. But after that, for 2^5, 2^6, 2^7 and 2^8, the last digits will be again 2, 4, 8 and 6 respectively.



So, let us just write down the possible last digits of the powers (greater than 1) of the numbers in the pattern in which they occur.

Last digit of the power of:

⇒ 2 → 2, 4, 6, 8

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$\Rightarrow 3 \rightarrow 3, 9, 7, 1$

$\Rightarrow 4 \rightarrow 4, 6$

$\Rightarrow 5 \rightarrow 5$

$\Rightarrow 6 \rightarrow 6$

$\Rightarrow 7 \rightarrow 7, 9, 3, 1$

$\Rightarrow 8 \rightarrow 8, 4, 2, 6$

$\Rightarrow 9 \rightarrow 9, 1$

$\Rightarrow 0 \rightarrow 0$

$\Rightarrow 1 \rightarrow 1$

From here, we can conclude that after at most every 4th power of any number, the last digit starts repeating itself. Let's take some examples to understand it better.

E.g. Find the remainder when 7^{267} is divided by 10.

Sol: Here, we just have to find the last digit of the number. We can see that the last digit of power of 7 starts repeating after every 4th power. So, we can simply divide the power with 4 and find the remainder.

So, $267 = 4 \times 66 + 3$

So, the remainder is 3. Now, the last digit of 7^3 is 3. So, the remainder will be 3. (**Ans.**)

E.g. Find out the last digit of the equation $222^{555} + 555^{222}$.

Sol: We just have to find out the last digits of 2^{555} and 5^{222} and then we will add them.

Now, the last digit of any power of 5 will be 5.

And, the last digit of any power of 2 starts repeating after every 4th power.

And, $555 = 4 \times 137 + 3$.

So, $2^3 = 8$

Now, $5 + 8 = 13$

So, the last digit will be 3 (not 13, remember). (**Ans.**)

E.g. Find out the last digit of equation $357^{223} \times 864^{1446} \times 2509^{45786} \times 5^{12346}$.

Sol: Here, we can see that the equation has 4^{1446} and 5^{12346}. We know that, the power of an even number is always even and any power of 5 will always give 5 as the last digit.

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And 5 × (any even number) has always 0 in its last digit.

So, the answer is 0. (**Ans.**)

Now, let's discuss how to find the last two digits of numbers.

Last two Digits

We also saw in exams, the questions like, what will be the last two digits of 21^{137} or what will be the remainder if 21^{137} is divided by 100. See, we can clearly say that since the unit digit is 1, the last digit of the answer will be 1. But we are not sure about the second last digit.

So, let's check the last two digits of the power of the number containing unit digit in base as:

For 0:

For any number ending with 0, like 70^{126}, 20^{35}, 3000^{1235}, 76450^{1235}, the last two digits will always be **00**.

For 5:

For the number ending with 5, it has two cases:

i) The power and the second last digit of base is odd → **75**

ii) Otherwise → **25**

E.g. Find the remainder when $(275)^{123}$ is divided by 100.

Sol: Here, we have to calculate the last two digits of the number.

Since the second last digit (i.e. 7) and the power (i.e. 123) both are odd, so the last two digits will be 75.

So, the remainder will be 75. (**Ans.**)

E.g. Find the last two digits of $(385)^{167}$.

Sol: Since, the second last digit of base is even, so the last two digits will be 25. (**Ans.**)

For 1:

We know that, for number ending with 1 with a given power, the last digit will always be 1. Now, the second last digit will be the last digit of the product of the second last digit of base and last digit of power.

i.e. for number like $(xy1)^{abc}$, the second last digit = last digit of (y × c)

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E.g. Find the last two digits of $(381)^{167}$.

Sol: The last digit of $(381)^{167}$ will be one. Now,

Second last digit = Last digit of [(Last digit of power) × (Second last digit of base)]

⇒ Second last digit = Last digit of (7 × 8) = 6

So the last two digits are 61. (**Ans.**)

For 3, 7 and 9:

We know that every fourth power of 3 and 7 is 1 and every second power of 9 is 1. So, we have to convert the unit digit into 1, then apply the trick we have discussed "**For 1**".

E.g. Find the last two digits of $(83)^{382}$.

Sol: (We'll mention last two digits only, so be aware)

⇒ $(83)^{382} = (83^4)^{95} (83^2)$

Now, $83^4 = (83^2) (83^2) \rightarrow (89)^2 \rightarrow 21$

⇒ $(21)^{95} (83)^2 = (01) (89) = 89$

So, the last two digits will be 89. (**Ans.**)

E.g. Find the last two digits of $(47)^{97}$.

Sol: $(47)^{97} = (47^4)^{24} (47)$

Now, $47^4 = (47^2)^2 \rightarrow (09)^2 = 81$

⇒ $(81)^{24} (47) \rightarrow 21 \times 47 \rightarrow 87$

So, the last two digits will be 87. (**Ans.**)

For 2, 4, 6 and 8:

There are only 4 things to remember (apart from all values from 2^1 to 2^{10}, which we have to learn).

Last two digits of $2^{10} = 24$

Last two digits of $24^{odd} = 24$

Last two digits of $24^{even} = 76$

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Last two digits of 76^{any} = 76

For, 4, 6, 8 → We have to split the number as a multiple of 2. Then we will apply all the previous tricks.

E.g. Find the last two digits of $(98)^{84}$.

Sol: $98^{84} = 2^{84} \times 49^{84}$

$\Rightarrow (2^{10})^8 (2^4) \times (49^2)^{42} \rightarrow 76 \times 16 \times (01)^{42} \rightarrow 16 \times 01 = 16$

So, the last two digits will be 16. (**Ans.**)

E.g. Find the remainder when $(158)^{432}$ is divided by 100.

Sol: $(158)^{432} \rightarrow (58)^{432} = 2^{432} \times 29^{432} = (2^{10})^{43} \times 2^2 \times (29^2)^{216}$

$\Rightarrow 24 \times 4 \times (29^2)^{216}$

$\Rightarrow 96 \times (41)^{216} \rightarrow 96 \times 41 \rightarrow 36$

So, the remainder will be 36. (**Ans.**)

[Note: Last two digits of $k^2 = (50 - k)^2 = (100 - k)^2 = (100 + k)^2$]

Let's see one more question and we will bind up after that.

E.g. What is the remainder when 767^{1009} is divided by 25?

Sol: Here, we know that 25 = 100/4. So let's just find out its remainder when divided by 100 i.e. its last two digits.

$\Rightarrow (767)^{1009} \rightarrow (67)^{1009} = (67^2)^{504} (67) \rightarrow (89^2)^{252}(67) \rightarrow 21^{252} \times 67 \rightarrow 41 \times 67 \rightarrow 47$

So, the last two digits are 47.

Now, $47 = 25 \times 1 + 22$

So, the remainder is 22. (**Ans.**)

We will not find these type of questions in the WBCS Exam. But this trick will help you to solve other questions regarding remainders and all which we will discuss in our upcoming blogs.

So, in conclusion, 'Learn to Find the Last Digits of Numbers' emerges as a transformative tool for aspirants gearing up for the WBCS Exam. This resource, meticulously designed to decode numerical intricacies, promises to be a game-changer in the pursuit of success. As candidates delve into practical examples and master essential techniques, they not only enhance their problem-solving efficiency but also fortify their confidence in navigating arithmetic challenges.

The acquired proficiency in determining the last digits of numbers becomes a strategic advantage, offering a competitive edge on exam day. With a solid foundation laid by this invaluable resource, aspirants are well-equipped to face the arithmetic complexities of the WBCS Exam, ensuring a comprehensive grasp of numerical concepts and bolstering their overall readiness for success in this highly competitive examination.

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So, this is all for this blog. We will discuss the **Some theorems on Remainders** in our next blog of this 'Number System' blog series. Till then, keep practicing!

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